LETTERS TO THE EDITOR

To the Editor:

In "Experimental Study of Deep Bed Filtration: A Stochastic Treatment" (30, p., 267, Mar. 1984), Hsu and Fan have developed a mathematical model for predicting the evolution of the pressure drop in a deep bed filter as a function of operating time. The model was derived by means of a stochastic treatment of pore blockage of a filter, as suggested by Litwiniszym (1963) and led to the total number of pores that are blocked at time t as given in Eq. 15 of Hsu and Fan (1984): $E[N(t)] = n_0[1 - \exp(-kt)]$. The objective of this letter is to show another way for the above derivation which is much easier and simpler than that of Hsu and Fan. Of course, the final results from both approaches are identical.

Recently, Beeckman and Froment (1979, 1980, 1982) used a stochastic approach to describe events in a network of pores inside a catalyst. They dealt with the problem of catalyst deactivation by site coverage, pore blockage, and diffusion, but their general approach appears to be directly applicable to the problem of pore blockage in a filter.

Let αdt be the probability that a randomly chosen pore is blocked in the time interval dt. Based upon the multiplication law of probability, the probability $P(t+dt)$ that a pore is not blocked at time $t+dt$ equals the product of the probability of not being blocked at time t and of the conditional probability that the pore is not blocked in the time interval dt:

$$P(t+dt) = P(t)(1 - \alpha dt) = P(t) + (dP/dt)dt \quad (1)$$

so that the probability that a pore is still accessible at time t is given by

$$P(t) = \exp(-\alpha r) \quad (2)$$

With the total number of pores at time zero n_o, the total number of accessible pores at time t is described by

$$n(t) = n_0 \exp(-\alpha t) \quad (3)$$

Equation 3 gives the same result as Eq. 15 of Hsu and Fan. Following the developments of Hsu and Fan (1984), the final result is:

$$\Delta P(t)/L = (\Delta P/L)_o \exp(\alpha t) \quad (4)$$

which is Eq. 26 of Hsu and Fan.

It is seen that our Eq. 4 was derived in a few steps, without functional treatments of differential equations such as the introduction of the probability generating function. Indeed, the Poisson distribution describes the problem well, in which expected mean frequency is small and the total number of events is large such as explained earlier.

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Reply:

We are happy to see that the model results in our paper, "Experimental Study Of Deep Bed Filtration: A Stochastic Treatment," have been confirmed independently by Nam and Froment by resorting to a simpler approach than that employed by us. The major contribution of our work, however, is in experimental verification of the model. It is preferable to use an apparently cumbersome, but more formal, approach of stochastic processes employed by us. Such an approach is applicable to a wide range of chemical engineering systems (McQuarrie, 1963; Litwiniszyn, 1963, 1966, 1968; Legg, 1983; Fan et al., 1982; Nassar et al., 1983; Fan et al., 1985) and can be extended to handle nonlinear phenomena through approximation such as system size expansion techniques (van Kampen, 1981; Gardiner, 1984). Moreover, the approach of Nam and Froment is suitable only for the case where the intensity of transition, λ_n, takes the form

$$\lambda_n = \alpha(n_0 - n), n = 0, 1, 2, \ldots, n_0 \quad (1)$$

and is based on the assumptions that:

1. The blockage mechanism of each pore is identical and independent.
2. The intensity of blockage for each pore is independent of time as well as the total number of open pores.

Under these assumptions,

$$Pr[\text{an open pore is blocked at time } t] = 1 - e^{-\alpha t} \quad (2)$$

If there are n_0 open pores initially, the number of blocked pores, n, at time t follows the binomial distribution (Mood et al., 1974):

$$P_n(t) = \binom{n_0}{n} (1 - e^{-\alpha t})^n (e^{-\alpha t})^{n_0 - n} \quad (3)$$

As noted in our paper, not only the mean but also the variance of the number of blocked pores at time t can be recovered, respectively, as

$$E[N(t)] = n_0(1 - e^{-\alpha t}) \quad (4)$$

$$\text{Var}[N(t)] = n_0(e^{-\alpha t})(1 - e^{-\alpha t}) \quad (5)$$

NOTATION

$E[N(t)]$	= expected value of the random variable $N(t)$
$N(t)$	= random variable which describes the number of blocked pores at the moment t
n	= values which may be assumed by the random variable $N(t)$
n_0	= total number of open pores susceptible to blockage
$P_n(t)$	= probability that there are n pores blocked at the moment t
t	= time
$\text{Var}[N(T)]$	= variance of the random variable $N(t)$

Greek Letters

α	= proportionality constant defined in Eq. 1, the blockage constant
λ_n	= intensity of transition

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To the Editor:

In the paper entitled "Comments on Effective Diffusion and Conduction in Two Phase Media: a Unified Approach" (29, p. 846, May, 1983), Hsueh-Chia Chang aims to unify three approaches for a two phase media—the generalized function approach, multipoles approach and volume-averaging approach. However, the results of this paper are alarming. The expressions 6, 9 and 11 indicate that effective conductivity (steady state) can be replaced by effective diffusivity (transient state) of the two-phase media. Wiener bounds considered for effective thermal conductivity (λ_e) of a two-phase media are:

$$\lambda_e = \lambda_s\phi_s + \lambda_f\phi_f \text{ (for parallel slabs)} \tag{1}$$

and

$$\lambda_e = \left(\frac{\phi_s}{\lambda_s} + \frac{\phi_f}{\lambda_f}\right)^{-1} \text{ (for series slabs).} \tag{2}$$

where λ_s, λ_f are the thermal conductivities of solid and fluid components, while ϕ_s and ϕ_f are the volume fractions of solid and fluid components.

According to the author's mathematical outcome expressions 1 and 2 can be changed to express effective thermal diffusivity (α_e) as:

$$\alpha_e = \alpha_s\phi_s + \alpha_f\phi_f \text{ (for parallel slabs)} \tag{3}$$

$$\alpha_e = \left(\frac{\phi_s}{\alpha_s} + \frac{\phi_f}{\alpha_f}\right)^{-1} \text{ (for series slabs)} \tag{4}$$

where α_s and α_f are the diffusivities of solid and fluid components. But using homogeneity criterion (Hori, 1973) for random and dispersed media, the effective diffusivity of a homogeneous isotropic media can be defined similar to the diffusivity of a single-phase media ($\alpha = \lambda/\rho c$), i.e.

$$\alpha_e = \frac{\lambda_e}{\rho_e c_e} \tag{5}$$

where $\rho_e c_e$ is the effective volume sp. heat of the media.

Therefore, using expression 5, the Wiener bounds yield the effective thermal diffusivity as:

$$\alpha_e = \alpha'_s\phi_s + \alpha'_f\phi_f \text{ (for parallel slabs)} \tag{6}$$

$$\alpha_e = \left(\frac{\phi_s}{\alpha'_s} + \frac{\phi_f}{\alpha'_f}\right)^{-1} \text{ (for series slabs)} \tag{7}$$

where

$$\alpha'_s = \frac{\alpha_s\rho_s c_s}{\rho_e c_e}$$

$$\alpha'_f = \frac{\alpha_f\rho_f c_f}{\rho_e c_e}$$

$\rho_s c_s, \rho_f c_f$ = volume specific heats of solid and fluid phase.

Using expression 5, other effective thermal conductivity expressions can also be changed to estimate effective thermal diffusivity replacing λ by α' but not by simple α.

However, the validity of the expression is examined by comparing its results with experimental results. Tables 1 and 2 for estimated effective thermal diffusivity (using different effective conductivity expressions) values through replacement of λ by α and λ by α' indicate that replacement of λ by α' gives better results. Therefore, there is a disagreement to the author's unified approach results.

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TABLE 1. ESTIMATION OF α_e($\times 10^{-3}$ $cm^2 \cdot s^{-1}$) OF TWO-PHASE SYSTEMS CONSIDERING SIMPLE REPLACEMENT OF λ BY α'

Systems (Preston, 1957)	Porosity ϕ_f	Expt. Value	Maxwell's Relation	Litchtenecker's Relation	Landau's Relation	Weiner Model		Hashin Shtrikman's Relation		Woodside and Messmer Relation
						Series	Parallel	Lower	Higher	
Glass-Water	0.408	3.07	2.95	3.47	3.78	2.7	4.33	3.44	3.95	3.92
Glass-Glycerine	0.428	2.94	2.6	3.19	3.53	2.38	4.12	3.13	3.75	3.67
Silica-Water	0.430	8.4	9.4	12.1	20	3.33	34.3	6.47	28.6	12.62
Silica-Glycerine	0.424	8.08	8.68	11.7	19.9	2.98	34.5	5.87	28.8	11.69
Stainless-steel-Water	0.501	7.9	2.8	9.2	15.7	2.88	29.7	5.3	24.2	10.39
Stainless-steel-Glycerine	0.502	7.41	2.4	8.62	15	2.53	29.5	4.7	24	9.42
Glass-Air	0.426	2.1	11	29	49	11	101	18	81	95
Silica-Air	0.408	2.35	86	103	111	85	129	103	117	90.3
Copper-Water	0.384	18.2	190	88	319	3.85	690	8.53	580	20.57
Copper-Glycerine	0.386	15.9	172	82	314	3.36	688	7.45	577	18.08
Stainless-steel-Air	0.502	1.44	93	116	126	93	144	116	414	140.5
Aluminium-Air	0.555	3.46	319	411	442	340	505	414	464	465
Copper-Air	0.388	1.73	517	606	666	448	775	593	708	1030

TABLE 2. ESTIMATION OF $\alpha_e (\times 10^{-3}\ cm^2 \cdot s^{-1})$ OF TWO PHASE SYSTEMS CONSIDERING SIMPLE REPLACEMENT OF λ BY α'

Systems (Preston, 1957)	Porosity ϕ_f	Expt. Value	Max. well's Relation	Litchteneker's Relation	Landau's Relation	Resistor Model		Hashin Shtrikman's Relation		Woodside and Messmer Relation
						Series	Parallel	Lower	Higher	
Glass-Water	0.408	3.07	3.06	2.97	3.05	2.78	3.16	3	3.06	3.09
Glass-Glycerine	0.428	2.94	3	2.91	2.99	2.73	3.1	2.94	3	3.04
Silica-Water	0.430	8.4	20	11	15.8	4.67	23.7	8.27	20.16	13.62
Silica-Glycerine	0.424	8.08	20.7	11	15.8	4.07	24.4	7.44	20.66	12.78
Stainless-steel-Water	0.501	7.9	22.7	9.35	15.2	3.16	27.8	5.74	22.7	10.89
Stainless-steel-Glycerine	0.502	7.41	22.4	8.57	14.6	2.68	27.6	8.5	22.7	9.65
Glass-Air	0.426	2.1	5.4	2.2	3.72	0.583	6.42	1.14	5.37	2.25
Silica-Air	0.408	2.35	49	8.12	27	0.523	59	1.13	49	2.73
Copper-Water	0.384	18.2	540	86.5	301	4.42	641	9.7	536	23.48
Copper-Glycerine	0.386	15.9	540	87	300	4.32	645	9.5	541	23.07
Stainless-steel-Air	0.502	1.44	46	3.98	19.5	0.279	58	0.55	46	1.34
Aluminium-Air	0.555	3.46	666	12.97	201.7	0.427	850	0.81	665	1.9
Copper-Air	0.388	1.73	877	43.58	452.3	0.309	1120	0.688	938	1.69

Reply

Profesor R. G. Sharma has compared our prediction of effective thermal diffusivity ($\alpha = \lambda/\rho C_p$) to experimental data and found that better agreement can be achieved if λ, the thermal conductivity, is averaged instead of α. After examining our manuscript in detail, we found his observation to be absolutely correct and located the error in our derivation. First of all, our results for effective diffusion is still valid. For conduction, however, the governing equation should be:

$$\rho C_p(x) \frac{\partial u}{\partial t} = \nabla_x \cdot \lambda(x) \nabla_x u \quad (1)$$

Unfortunately, in Eq. 1 of our paper, in an attempt to unify the analysis for both conduction and diffusion, we chose to study:

$$\frac{\partial u}{\partial t} = \nabla_x \cdot \alpha(x) \nabla_x u \quad (2)$$

which is incorrect since $\rho C_p(x)$ is a function of position in our study of two-phase media. This error can be made more explicit by integrating Eq. 2 across the phase boundary to yield:

$$\alpha \nabla_x u \cdot \mathbf{n}|_{\text{phase 1}} = \alpha \nabla_x u \cdot \mathbf{n}|_{\text{phase 2}} \quad (3)$$

which is in disagreement with the correct heat flux balance across phase boundaries:

$$\lambda \nabla_x u \cdot \mathbf{n}|_{\text{phase 1}} = \lambda \nabla_x u \cdot \mathbf{n}|_{\text{phase 2}} \quad (4)$$

Note, however, as λ_1/λ_2 or $\alpha_1/\alpha_2 = \beta$ approaches infinity or zero, the two formulations are in exact agreement. This explains why our theory agrees so well with experimental data for periodic cylinders at $\beta = 0$ and 160 in Figures 5 and 6 of our manuscript.

Fortunately, there is a simple remedy to our error, other than the unlikely assumption that ρC_p is equal in both phases. If one examines only steady-state conduction or if one leaves the capacitance term ρC_p at its proper location shown in Eq. 1, all of our derivations and results are valid if one replaces α by λ (or simply defining α as the thermal conductivity). In the transient case, the effective equation would become:

$$\langle \rho C_p \rangle \frac{\partial u_0}{\partial t} = \nabla_x \cdot \mathbf{A} \cdot \nabla_x u_0 \quad (5)$$

which replaces Eq. 11 of our manuscript. Hence, the only necessary modification is exactly the one suggested by Professor R. G. Sharma—average λ or $\lambda/\langle \rho C_p \rangle = \alpha'$ instead of α. In fact, the excellent agreement to experimental data of nonconvective systems shown in Professor Sharma's letter indicates that the theory is still robust after the above modification.

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